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Instructional Design Perspectives on Mathematics Education With Reference to Vygotsky's Theory of Social Cognition

Brent G. Wilson
School of Education and Human Development at the University of Colorado Denver, brent.wilson@ucdenver.edu

James L. Teslow

Lyn Taylor

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Educational researchers take different positions on the question of instructional design for different content domains. Some say the content disciplines are essentially unique; teaching strategies that work in social studies, for example, will not work in mathematics. Following this view, the development of teaching models should be unique to the content domain. The field of mathematics education is predicated on the view that content-specific instructional strategies are essential. Others believe that we can develop a set of generic teaching methods that can be selectively used in the teaching of different content domains (Reigeluth, 1983, 1987).

A moderate position affirms the value of both generic and content-specific research and strategies. This moderate approach seems to be less dogmatic and more promising in the long run, granting value to various forms of knowledge about teaching. Because content domains draw on common learning mechanisms, there are likely some models and strategies that would be appropriate across domains. Even so, instructional research in specific domains can complement whatever generic understanding we have of instructional processes. Particularly valuable activities include conducting deep content and cognitive task analyses, testing out specific teaching strategies, and examining learner differences in specific learning environments.

The field of instructional design is based on the notion that generic strategies of instruction can have value across any content domain (Reigeluth, 1987; Wilson, Jonassen, & Cole, in press). The primary aim is to distill what we know about learning and instruction based on current research and theory, then to develop prescriptive models and strategies for teaching . Simon (1983) suggests that "design sciences" such as instructional design can articulate explicit principles of design that can be useful in solving real-life problems. Viewed in this way, instructional design is more a technology than a pure science.

The purpose of this article is to offer some perspectives on mathematics education from an instructional design viewpoint. We do this in a somewhat eclectic fashion, beginning with an overview of the ideological "paradigm wars" within the instructional design community. Alternative philosophies of mind, including Vygotsky's (1978) emphasis on the social origins of cognition, have implications for the teaching of mathematics, as well as for instructional design generally (e.g., Lave, 1988). In the latter half of the paper, we assume Jim's voice as he provides a personal narrative of the importance of social and cultural realities as a professional engineer. We conclude with some recommendations for the instructional design of mathematics education curricula that are consistent with a Vygotskian framework.

The authors of this article have some background in mathematics, but claim no expertise in the specific field of mathematics education. Because of our different background, some of what follows will probably sound old; other parts will be familiar but perhaps presented with a different twist; hopefully something will be new and helpful. We believe that researchers need to cross disciplinary boundaries more than we usually do, addressing new audiences and responding in kind.

Paradigm Wars in Instructional Design

The field of instructional design is undergoing considerable upheaval presently as theorists attempt to move from behaviorist roots toward cognitivist and postmodern interpretations of their practice. Recently, two issues of *Educational Technology* (May and September 1991) were devoted to constructivism (see discussion below). The growing pains, including resistance from the Old Guard, are evident in those issues. However, we suspect that instructional designers are not alone in trying to come to terms with constructivism and other emerging models of cognition. Virtually every educational area is wrestling with similar issues as we all try to adjust to advances in cognitive science and philosophy of mind.

Emerging from a behaviorist base in the 1950s and 60s, instructional design has worked to accommodate information-processing models of cognition. Gagné, instructional design's leading theorist, modified his taxonomy of learning outcomes and conditions of learning to suit an information-processing model (Rickards, 1978). Recently, constructivist ways of viewing cognition have had considerable impact (Bednar, Cunningham, Duffy, and Perry, 1991). Connectionist and postmodern frameworks have also begun to receive attention (Wilson & Cole, 1991a). Table 1 lists these several learning theories that have influenced instructional design over the years. Below, we briefly review the theories in the historical order of their influence.

Behaviorism. Behaviorism comes in different forms. The radical behaviorism of B. F. Skinner is an extreme form, yet more moderate, subtle forms remain common in education. Results-oriented, objectives-based training shows a behaviorist bias. Standardized, objective testing also has a behaviorist, positivist foundation (Shepard, 1991). Instructional designers' insistence on defining instruction in terms of behavioral objectives and behavioral performance measures is traceable to behaviorism's continuing influence on the field.

Information processing. The key metaphor for information-processing models of cognition is the human mind as a computer. Information-processing models suggest that people "store" concepts and rules in a "place" called memory through a vast network or schema of interrelated symbols, propositions, and knowledge components. The metaphor is pervasive: most educators think of cognitive psychology and information-processing models as almost synonymous. Distinctions like short- vs. long-term memory, declarative vs. procedural knowledge, and semantic encoding vs. retrieval have become part of our commonplace understanding (E. Gagné, 1985).

Behaviorism
Mind as black box
Knowledge as behavior
Learning as reinforcement
Teaching as control of environment
Information Processing
Mind as computer
Knowledge as object
Memory as place containing objects
Expert performance as rule following
Learning special case of performance
Learning as acquisition of rules, concepts, procedures
Teaching as mapping expert's cognitive map onto learner
Constructivism
Mind as inner representation of outer reality
Knowledge as residing in the individual mind
Meaning as internally constructed
Individual reflection/abstraction as primary
Learning and teaching as negotiated construction of meaning
Connectionism
Mind as brain (mind/body dualism eliminated)
Mind as material machine (AI or natural intelligence)
Knowledge as pre-symbolic, pre-representational
Representation as a special case of knowledge
Knowledge as socially/environmentally distributed
Thinking as continual re-construction
Expert performance as pattern recognition, not rule following
Learning and performance as inseparable
Postmodernism
Mind as being-in-the-world
Life as a text; thinking/living as interpretation
Reality as inherently multi-perspectival
Individual as special case of group
Everyday life as primary
Individual as defined by social relations

Table 1. Learning theories underlying instructional design.

Constructivism. Constructivism is somewhat more difficult to nail down because the label covers a wide spectrum of beliefs about cognition (Jonassen, 1991). In general, old-style constructivists follow Piaget in emphasizing individual thinking and creation of meaning (e.g., Papert, 1988a). This is the notion of constructivism familiar to mathematics educators. Instructional designers generally tend to think of constructivism more broadly (and less tied to Piaget), leaving room for social cognition theorists such as Bruner and Vygotsky. David Merrill (1991), while not sharing constructivist beliefs, does a good job of summarizing the constructivist framework:

--knowledge is constructed from experience;

--learning results from a personal interpretation of knowledge;

--learning is an active process in which meaning is developed on the basis of experience;

--learning is collaborative with meaning negotiated from multiple perspectives;

--learning should occur (or be 'situated') in realistic settings; and

--testing should be integrated into the task, not a separate activity.

Constructivism tends to be more holistic and less mechanistic than traditional information-processing theories (Cunningham, 1991). People make sense out of their world by taking in information from the environment and assimilating it into their preexisting schemas and understandings (Bransford & Vye, 1989). Children undergo conceptual change by directly confronting misconceptions (Wilson & Cole, 1991b).

Connectionism. Connectionism reflects research in cognitive science-artificial intelligence, neurology, and computer science-aimed at modeling neural processes. The mind is portrayed as a neural net, wherein the "knowledge" resides entirely in the patterns and relations among binary neurons. Neurons themselves have no content; at any moment they are in one of two states: excitatory or inhibitory, on or off. Thus instead of a semantic network of meaningful rules and concepts, intelligence *emerges* out of patterns of "dumb" neurons through the strengths of associations based on experience (Papert, 1988b). Neural nets are sub-symbolic-meaningful symbolic units may be consciously constructed and reflected on by a person but do not account for basic processing. In like manner, neural nets are pre-representational; learners may construct a conscious representation or image of something they believe to be "out there;" however, the network need not reflect external reality. Connectionists have aligned themselves with the "situated cognition" movement (Brown, Collins & Duguid, 1989), asserting that because cognition depends on our experience base, cognitive apprenticeships and other "authentic" learning methods are recommended (Clancey, 1992).

Postmodernism. An important theory and accompanying set of methodologies has emerged from continental philosophy and critical analysis within the humanities tradition. Hermeneutics, semiotics, and phenomenology are terms associated with this approach (Hlynka & Belland, 1991). These approaches share a common claim that thinking is primarily an interpretive activity. Postmodern theorists reject the correspondence theory of truth that defines truth as an accurate internal reflection of the external world. Thinking is not a matter of creating internal representations of real things out in the world. The question is not how do I *represent* an external world truthfully, but rather how do I *interpret* these interactions in a way that is truthful to them? Instead of assuming that individual minds are a starting point for understanding the human condition, postmodern theorists state that the unquestionable given in life is that we are already in the world, interacting with other people and situations, already involved in relationships and commitments (Faulconer & Williams, 1990).

Cognition is seen by some as an internalization of social interaction (Vygotsky, 1978; Wertsch, 1985, 1991). Vygotsky was concerned with culturally situated learning, believing that educational interactions reflect the surrounding culture. Yet he also demonstrated that once individuals are operating at higher concept levels, such as understanding scientific concepts, students may think abstractly.

Postmodern models of mind agree in many ways with connectionist models. Both approaches are non-representational, both agree that traditional Cartesian dualism is inadequate. Both agree on the importance of tacit knowledge and cultural/environmental variables. Both approaches suggest similar instructional implications, such as learning environments and cognitive apprenticeships. They part ways, however, in their attitude toward the nature of intelligent activity. Connectionists are actively working to develop computer-based neural networks that embody intelligence (Rumelhart, McClelland, and the PEP Research Group, 1986); postmodern theorists argue that computers can never capture true human-like intelligence (Dreyfus, 1979; Winograd & Flores, 1986).

It is clear that recent models of cognition are challenging traditional notions of learning and teaching. However, for those of us raised on objectivist, representational models of mind, old habits tend to die hard. It seems counterintuitive to not think of a real, true world out there that our minds are trying to capture. Many people feel secure when their disciplines are clearly and explicitly defined as a set of rules and systematic principles. People like recipes to guide their actions (Putnam, 1991).

Instructional implications of connectionist and interpretative approaches have not yet been thoroughly worked through. At a time of such basic re-thinking about the nature of cognition, it is hard to be dogmatic about what teaching strategies comprise the "optimal" instructional design in any subject matter. Perhaps the main lesson for now is that the discussion below should be read with a certain degree of skepticism. Our knowledge base in cognition and instructional design really is fragile, depending on a shifting foundation that will likely continue to change in the years to come.

How do the ideological disputes within instructional design relate to mathematics education? Mathematics education, like instructional design, is a derivative, applied field rather than a basic scientific field. Mathematics educators, like instructional designers, depend on cognitive science and other theoretical foundations to provide grounding for specific models and strategies. Mathematics education, like instructional design, is constantly in a state of re-construction as it re-examines its theoretical underpinning in light of new understandings in philosophy and cognitive science. As sympathetic observers, we ask how current theories such as connectionism and postmodernism have affected mathematics education? We hope that someone will pick up the work and articulate implications of these theories for the practice of mathematics education.

Teachers and Ideology

There is another moral to draw out of this discussion. Seeing the paradigm wars within a discipline clarifies the ideological nature of our fields. Instructional design and mathematics education are both highly value-laden enterprises. People have stakes in different outcomes, different theories and models. Imposing an ideology on people is a risky business, in politics as well as schools. An illustration may help to clarify this point:

Case 1: Brent's account of his son's Algebra teacher. Mr. Hoffman is a 7th and 8th grade algebra teacher in Boulder, Colorado, where my son Seth attends school. Mr. Hoffman is-to use the popular parlance-a very left-brained, nerdy kind of guy. Somehow tied up with his personality, or his personal belief system, is an approach to mathematics instruction that emphasizes repetition of skills, mastery of discrete rules, rigorous practice and testing of predefined skills. I am told he gets good results on the standardized tests.

I have talked with Mr. Hoffman enough to be convinced that there is no way that I could get him to change his teaching style. I think Seth would benefit more from a teacher who emphasized meaningful connections, creative solutions, open-ended problem-solving, but it won't be Mr. Hoffman.

After two years with Mr. Hoffman, Seth was looking forward to a more creative, open type of teacher for next year; Mrs. Blalock had a reputation for being the best, most creative teacher in the area. Unfortunately, the school changed from a junior high to a middle school for next year, and Mrs. Blalock was moved up to the high school, where Seth won't be able to take classes. He is frustrated.

Seth had Mr. Hoffman for two years and was fortunate to do well in his class (although we'll never know what he missed). Seth has a friend, though, whose learning needs directly clashed with Mr. Hoffman's teaching style. Nathanael is among the brightest 8th grade mathematics students around, placing first in two different state mathematics competitions last year. But Nathanael is very holistic and intuitive, with a need to understand the concept behind the procedure. His self-concept suffered through the B's and C's with Mr. Hoffman, and he struggled to understand the content.

We all know Mr. Hoffmans, somewhere in our past or our present. Labeling him a poor teacher oversimplifies. His students do learn; they do well on standardized tests. Mr. Hoffman's case is presented because it illustrates how ideology and teaching style are often mixed up in the person. Just as the country accommodates a wide array of political beliefs, there needs to be some accommodation within school systems of teachers with different teaching philosophies, personalities, and aptitudes. Mr. Hoffman will probably never convert to Vygotskian principles.

On the other hand, he may. Here is a second example of teachers and ideology. We are both engaged in research in an elementary school that is reported below.

Case 2: Changes in teachers following introduction of technology into a school. Peakview Elementary School is a new school in Colorado's Cherry Creek School District. After teachers were hired, the principal and the district decided to place 4-6 networked Macintosh LCs into each classroom. A computer coordinator supervised initial teacher training on the Mac-primarily Microsoft Works-in the spring. Teachers then took Macs home for six weeks in the summer to play with. Many teachers reported getting help from their kids at home.

In the fall, teachers incorporated the computers into their everyday teaching. We were part of a team contracted with the district to evaluate the impact of the technology on the school. We surveyed and talked with every teacher, as well as large samples of students and staff members. We observed and videotaped classrooms. We read teachers' weekly use logs and diaries. We examined student work samples.

As expected, students loved the technology. More surprisingly, from our extensive teacher interviews and surveys, we couldn't find one teacher who wasn't also enthralled by the technology. We couldn't find one teacher who displayed any resistance to the innovations in their everyday teaching routine. Recall that these teachers were not hired for their technological expertise; many came on very hesitant and skeptical, but each one of them accommodated their teaching activities to the new technology, increasing students' use of word processing for revising and editing written work, research using CD-ROM and laserdisc, *HyperCard* and graphics programs, and other instructional software. Each one of them became "converted" to new ways of teaching and using technology.

This seems to be a counter-example to Mr. Hoffman's apparent intransigence. Teachers who never imagined they would use computers suddenly find themselves making continual, daily use of them, with accompanying changes in teaching strategy. In less than a year, major changes were forged in everyday teaching activities and in teachers' belief systems. And without a single exception, teachers were positive about the changes.

So where are we with respect to ideology and teaching? Can teachers make sweeping changes in their teaching orientation? Under certain conditions, teachers can undergo substantial shifts in the meaning they attach to their teaching activities. In fairness, our Peakview research did not directly assess teachers' underlying philosophies. Some teachers seemed to have undergone radical changes in their beliefs about teaching, but then other teachers very likely preserved prior conceptions. Perhaps teachers should be encouraged to constantly reflect on and examine their beliefs about teaching, and be open to change and innovation to make their practice more consistent with their beliefs (Schön, 1987). Having teachers who are reflective and open to change will benefit students, since the teacher variable is extremely important for students' success in and attitudes toward mathematics (e.g., Midgley, Feldlaufer, & Eccles, 1989).

Mathematics and Sensemaking

Presented below are a few key concepts characteristic of a constructivist framework.

Memories of Early Schooling

Jim has an engineering background; we draw upon his professional experience later in the article. Jim's recollection of his early schooling is probably fairly typical among today's engineers.

Case 3: Jim Teslow's experience in grade school. Most engineers will tell you that, based primarily on abilities identified at an early age, they were nurtured for the profession by encouraging teachers. This was true in my case. In the 1950's and early 60's, the national mandate for public schools was to produce scientists and engineers that would keep the country abreast or ahead of the USSR in technology. With the launch of Sputnik, the welfare and security of the country suddenly depended on more than military aegis. Elementary school teachers were called upon to identify and promote young candidates for the cause.

I can remember being given extra work in primary grades. For example, during many months in fifth grade I searched through a book about South America looking for phrases to fill in blanks in a workbook. I remember a feeling of satisfaction that was related to the *production* aspects of the task. There was very little challenge involved, since the workbook sentences were exact replicates of the text entries. It was simply a matching game, which I enjoyed because I could do lots of it, quickly. Was there a work ethic involved that had been inculcated? Perhaps I received praise from my teacher for accuracy and speed. Whatever the source of the positive feelings for my efforts, I was busily constructing a set of beliefs and values that would result in successful performance in all subsequent procedural activities-especially in mathematics and science courses. The penchant for finding information served me well, right through senior year Calculus and Physics, by which time I could rightfully claim to be a *find-and-plug-in* expert.

Schoenfeld (1991) has surveyed research that shows the negative consequences of traditional mathematics teaching in the schools. Several examples are presented where students, steeped in the everyday procedure-oriented practices of the classroom, develop mathematical "nonreason" (i.e., a willingness to engage in activities that don't make sense) and suspend sensemaking. The most glaring feature that one is struck with when reading Schoenfeld's survey of research is the tendency for students to use all available data in the problem statement, regardless of its relevance. For example, when first- and second graders are asked, "If there are 26 sheep and 10 goats on a ship, how old is the captain?", seventy-six out of 97 students provided a numerical answer by adding 26 and 10 (p. 322).

Schoenfeld points out that hours of plowing through exercises on worksheets using a predetermined procedure, or working textbook problems that are neatly packaged with just the right data, effectively suspends the requirement that mathematics make sense in the classroom. Some teachers unwittingly assist their students in the suspension of sensemaking by providing them with rules to memorize and use. The *key word method,* for example, tells students that the word *left* is the clue that this is a subtraction problem (Schoenfeld, 1991). Look for the numbers and key words and you can probably obtain the correct answer without having to read the contrived story in between. Students learn from feedback on timed tests and nightly homework that the name of the game is to solve the problem, not to understand it.

Situated Cognition

The traditional textbook-type word problem asks students to solve supposedly real-life questions about people who do very unreal things (Brown, Collins & Duguid, 1989). For example, typical out-of-context word problems in high school mathematics courses usually involve problems that deal with contrived situations such as enclosing areas with a fixed amount of fencing material, traveling in perfectly straight lines at constant speeds, or filling leaking barrels with leaking buckets. These activities fail to provide activity relevant to successful performance outside the classroom. American schools are supposed to prepare us for life in an increasingly complex world that demands expertise. Given the reality that in-school experience oversimplifies the application of knowledge, how do adults become collaborative workers, motivated professionals and competent problem solvers? Collins, Brown & Newman (1989) assert that it is largely on-the-job or out-in-the-world apprenticeships that result in expertise. It's what people have done for years to adopt the behavior and beliefs of their peers.

Situated cognition is the notion of learning knowledge and skills in contexts that reflect the way the knowledge will be useful in real life (Collins, 1991; Lave & Wenger, 1991). The situated cognition framework maintains that cognition is not confined to the individual, but is coded by and connected to the activity and environment in which it was developed (Brown, Collins & Duguid, 1989). Thus, a key component of situated learning is that activity be authentic; that is, meaningful and purposeful.

The activities of a domain are framed by its culture (Lave, 1988; Wertsch, 1991). Their meaning and purpose are socially constructed through negotiations among present and past members (Brown, Collins & Duguid, 1989). Many of the activities students perform in school would not be recognized by practitioners of a culture that they are intended to represent; that is, they are not authentic activities.

In their presentation of cognitive apprenticeship as an alternative to conventional schooling, Collins, Brown & Newman (1989) propose the integration of realistic performance into instruction. By studying the way humans learn naturally, they equate learning with enculturation, the imitation of the behavior of a social group in accordance with its norms. Just as Jim learned the concepts, activities and culture of aerospace engineering by adopting, *in situ*, that community's socially constructed web of beliefs, students become enculturated into the ambient culture of a school. This situation would be fine if society's intention were to create expert school attenders.

Whitehead (1929) coined the term *inert knowledge* to describe material learned in this manner. Inert knowledge can be recalled when people are explicitly asked to do so, but is not used spontaneously in problem-solving, even though it is relevant. Such lack of transfer is troubling to educators. Many teachers are frustrated by the inert nature of much school knowledge.

Mathematics knowledge tends to be separated from its uses in the real world. Students rely on their school-culture expertise when asked to solve contrived problems. For example, they may use their knowledge of textbook structure ("I must use the Quadratic Equation because the problem is in that section"), rather than strategies or properties of the problem. Investigating this phenomenon, Schoenfeld (1985) studied the teaching of mathematical problem-solving to college students. Even though he could effectively teach algorithms (a sure-fire method that always leads to a solution of a particular problem), he found it difficult to teach the heuristic of when to apply the proper algorithm. It is the use of heuristics (rules-of-thumb that may solve a problem, but do not guarantee a solution) that distinguishes an expert from a novice. He found in his experiments that teaching a number of these heuristics, and how to apply them in different kinds of mathematics problems significantly increased students' problem-solving abilities.

Memorization vs. Tool Using

In spite of warnings (National Council of Teachers of Mathematics, 1989), current teaching often emphasizes memorizing rules (e.g., the formula for perimeter) and discrete pieces of knowledge (e.g., pi to six decimal places). If students forget the rule, they are unable to do the problem on their own. In addition to the rule, students need heuristics to help in knowing when to apply the rule. Unfortunately, students receive little practice applying heuristics in many classrooms. Often, problem solvers in school are required to demonstrate what they have learned by showing the instructor what they have temporarily remembered.

Ironically, outside of school, in professions such as engineering, architecture and law, reliance on memory is in fact discouraged! In the building of bridges, buildings, and precedent-based legal briefs too much is at risk to rely on what one can remember. Professional problem solvers are experts at acquiring just the required information, when it's needed (just-in-time), and using it for the achievement of a goal. Within arm's reach of real world experts are reference manuals, nomographs, charts, tables, blueprints, case studies, formulas, and facts designed to contain the declarative knowledge needed for a task.

Resnick (1987) has studied the wide use of cognitive and physical tools outside of school versus the emphasis placed on pure thought and symbolic rules in school. She points out that the generalities and symbolic rules used to solve problems in school are in direct contrast to the situation-specific competencies and context-based reasoning required to deal with society's economic realities. Using the piloting of a ship as an example of a situation where no single individual can accomplish the task alone, Resnick also makes the point that school settings are designed around the individual; thus, they are of little use in a world of socially shared tasks. The tool concept and its relation to authentic activity is not emphasized in school, resulting in a school environment that fails to provide activity relevant to successful performance outside the classroom. Thus, we graduate young adults who are experts at memorizing inert knowledge, performing simplistic, or brittle skills (Collins, Brown & Newman, 1989), and listening passively, instead of instilling in them the notion that much of the knowledge needed to perform tasks outside of school can be found in the tools of the workplace.

Of course some remembering is required in any authentic activity.

Efficient experts remember chunks of information that are used frequently and that are well-understood. Interrelationships build meaning. The new Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) place great importance on building connections, stating that teachers should "...represent mathematics as a network of interconnected concepts and procedures; and emphasize connections between mathematics, other disciplines, and daily life" (p. 89).

Out of School Realities-Jim's Example from Engineering

The sensemaking aspects of mathematics learning are essential if students are to successfully transfer their understanding to task requirements on the job. In this section, we use engineering to illustrate the importance of math making sense.

The social and political realities of a culture need to be taken into account by cognitive scientists modeling instruction on real life. In the field of engineering the enculturation process starts in a school setting-at the university. A typical engineering student on a large campus finds himself somewhat isolated-too practical to be respected by the pure sciences, too technical to be accepted as a scholar by the liberal arts or social sciences. Soon the student finds that almost all campus activities are centered around the School of Engineering, and that there are three fundamental belief systems that mantle all aspects of the culture. The first framework for the culture of engineering that will be discussed is its language-calculus. The second practice that shapes the profession is political and societal-maleness. The third characterization of the engineering environment is its unexpected individualistic rather that cooperative work styles.

In the remainder of this section, we assume Jim's voice in the first person as he recounts his engineering training and experience.

Calculus-A right of passage. First year engineering students who are unable to make the paradigm shift from discontinuous, disparate sources of rule-based mathematical information presented in many high schools to the spatial continua of calculus are soon pursuing other programs of study. For me (Jim), personal mathematical sensemaking started with applied calculus. The rich mental models resulting from the construction of meaning based on the infinitesimal, transformed mathematics into a belief system based on integrity and beauty. The real world takes on a smooth, inviting texture-functions become roller coasters rather than equations; volumes are swept out by hoops and wands, not mere calculations of base times height; fluid flow becomes an onslaught of arrows; and boundaries become approachable asymptotes.

At some point in the application of mathematics to the modeling of physical systems, this personalized sensemaking expands into a feeling of professional responsibility. The analyst might agonize for days over the source of an unanticipated minus sign. There is a reason for its presence-mathematics guarantees it. The cause is there waiting to be discovered. To ignore an anomaly would be unethical, an act of negligence, with possibly severe real life consequences.

After many years of problem solving and mathematical modeling, the engineer develops what is commonly referred to in the field as *engineering judgment*- a feeling in your bones, an uneasiness, perhaps, that something is not quite right, or alternatively, that it's safe to proceed. I have witnessed many instances where problems were not only solved, but anticipated or avoided by an expert's familiarity with a physical system-the professional's gut feel.

The Good-ole-boy Club. I have worked with several talented women engineers. I would venture to say that every one of them has been repeatedly asked, "Could you type this up for me?" Overcoming prejudice, discrimination, and the secretarial stereotype is bad enough, but another unfortunate and troubling bias has been studied recently. One of the factors that makes many women unwilling to pursue technological careers is the tacit assumption that they must *think* like men to succeed. Turkle and Papert (1991) studied women undergraduates in an introductory programming course who resisted the traditional, abstract and systematic (hard) techniques for controlling a program-planning, flowcharting, and modularization. They preferred a negotiational (soft) approach, which included concrete forms of reasoning, a closeness to computational objects, and often a bottom-up approach to development.

[Lisa] had been told that the "right way" to do things was to control a program through planning and blackboxing...Lisa recognized the value of these techniques-for someone else. She struggled against using them as the starting points for her learning. Lisa ended up abandoning the fight, doing things "their way," and accepting the inevitable alienation from her work. It was at this point that she called her efforts to become "another kind of person with the machine" her "not-me strategy," and began to insist that the computer is "just a tool." "It's nothing much," she said, "just a tool" (p. 164).

Turkle and Papert also point out that the social construction of science and engineering, with its canonical abstract and rule-driven style, is associated with male power and elitism. They call for the acceptance of multiple ways of knowing and thinking, as a challenge to the near-exclusive dominance of abstract, formal, and logical scientific thought. In their call for alternative voices in the world of formal systems, they look to the computer as a tool capable of accommodating diverse thinking styles. They argue for an "epistemological pluralism" that grants the legitimacy of a variety of problem-solving styles (Belenky, Clincy, Goldberger, & Tarule, 1986).

Increasingly, there is reason to be optimistic about the acceptance of alternate learning and thinking styles in science and mathematics workplaces. A number of changes indicate this:

--Object-oriented programming that replaces algebraic metaphors with natural interactive elements and congenial icons have changed the way we interact with computers.

--Graphic, iconic, and dramatic interfaces (Laurel, 1991) have become increasingly accepted.

--Computer-supported cooperative work (CSCW) has become an emerging field within human factors engineering (e.g., Dykstra & Carasik, 1991).

--Research on e-mail interactions and learning is increasing (Ahern & Repman, 1992; Drayton & Pfister, 1992).

--Better methods are being developed for visual data analysis and virtual reality (Kozma, 1992; Hardin, 1992).

These areas of nontraditional uses of computers for thinking and learning indicate that science and mathematics as disciplines and as work areas are rapidly changing, allowing for a greater variety of work strategies and cognitive styles.

I'd rather do it myself. In the engineering profession, some of the work is still based on rote activities. Modern entry-level engineers usually start out their careers inputting data into a computer model of a physical system. They become experts at manipulating the *black box* computer model to obtain solutions to problems, but never understand the physics involved. The engineer that designed and wrote the mathematical model understood the principles involved, but has, perhaps, long since retired.

It was the creation of a functioning analytical tool that brought about a feeling of independence and empowerment in my (Jim's) career. As part of this creative process, engineers are usually encouraged to develop a personal strategy. After the initial phase of getting accustomed to a workplace, the new engineer is many times given a long-term project to accomplish. In my case this took the form of writing a FORTRAN IV computer program from scratch that was to model the physics of a fluid subsystem. Time needed? Maybe three to six months. Guidelines? Just get it to work so its output can be compared to test data. Goal? Let's see how well mathematics can approximate a real, measurable, dynamic phenomenon. Given these kinds of individualized work assignments many engineers develop idiosyncratic work styles. For example, my personal style of programming is soft. I never flowchart or systematically plan my programs. I feel a closeness to the mathematical relationships that usually appear in the middle of the model. So, I'll start there and work outward to the input and output sections. I sometimes try out calculation loops as I go to see what happens locally. I seem to have an aversion to modularization, preferring instead to see the work as one entity. Many of my programming practices break the canonical rules-they are inefficient and nonhierarchical. They are unique but they work, and the task is accomplished.

Many engineers become isolated-experts at specialized tasks that utilize personally developed techniques. Collaboration and communication are not a typical engineer's strong points. In many competitive technical contexts there is also a tendency away from the *sharing* of sensemaking. If someone thoroughly understands a concept, why would there be a reluctance to share this understanding? In my work with engineers most admissions regarding this phenomenon involve risk. For example:

1. The personal interpretation or mental model may be considered primitive and unsophisticated, or

2. In a competitive workplace, job security may be enhanced if not too many other peers share your understanding.

However, in my experience, the most successful supervisors and managers, the most productive analysts, and the most creative designers were those with a bit of *teacher* in them-those willing to pass on knowledge and secrets of the trade.

In aerospace engineering coursework, it is not uncommon to encounter mathematical representations of physical concepts and theories that use up half the Greek alphabet in one expression. For example, I (Jim) studied the equations of rocket propulsion, aerodynamics, and gas dynamics for years before finally acquiring a strong, spatial, mental image of how a rocket engine works. One day, my first supervisor explained, "Look, it's really very simple. The thrust comes from three sources:

1. A pressure difference in the chamber caused by a hole in the bottom;

2. A reaction caused by the action of high-pressure gas rushing out the hole in the bottom; and

3. A nozzle that lengthens the hole in a special way so that the gas rushing out can push on something as it leaves."

Figure 1 Mental Model of a Rocket Engine.

I feel that I became an expert engineer when I was able to throw off the yoke of proceduralization, construct my own mental models, and share my sensemaking with others. In one career-defining moment, for example, I happened to be leaving a design meeting with a manager who mumbled that he will never be able to understand *mass fraction.* In rocketry, the definition of mass fraction is simple; however, engineering design work related to it is quite complex. It is a measure of design efficiency-the weight of the propellant (fuel) divided by the total weight of the stage (typical mass fractions are .85 to .90). Because the engines, tanks, and structure do not contain any inherent energy as does the propellant, the goal is to maximize propellant weight and minimize non-propellant weight. This relatively straightforward concept becomes a messy optimization problem among the various design disciplines. For example, one has to add structure and tankage to add more propellant; engines need to be modified for increased burn time; the launch pad needs to be modified for a longer stage, etc. The epitome of the concept is easily lost in the complex terminology of the optimization process.

"I like to think of a loaf of bread as having a very good mass fraction-close to 1.0," I said.

"You mean the bread is like fuel, and the wrapper is like structure...My God!" he said, "I get it! Thank you, that's just what I needed."

Because of my willingness to share my personal sensemaking, I found that I was increasingly sought out by upper management to perform an advisory (almost teacher-like) role. This visibility paid dividends-within two years I was promoted from staff engineer, to senior staff engineer, to manager.

Of course it is not fair to state that all of an engineer's expertise is developed through on-the-job apprenticeship, but Jim contends that most of his sensemaking arrived post-schooling. A recent annual convention of the American Astronautical Society devoted an unprecedented amount of its agenda to industry's frustration with the ineffectiveness of new engineering graduates. The business community is clearly tiring of its role as benefactor for the remediation of its workers. In this sense Jim's engineering experience corroborates the case made by the NCTM standards for improved methods of teaching mathematics from a sensemaking, problem-solving orientation.

Implications for Teaching Practice

The teacher certification programs of many older mathematics teachers taught the traditional behaviorist view of the student as an agent who is externally stimulated with patterns of reinforcement. Traditional teaching of problem solving relies on the *objectivist* view that knowledge is something that gets put into students. Meaning lies in the thing itself, independent of the experience of learners. Teaching equates to the inculcation of "the way the world is"-the unadorned entities, attributes and relations that exist in the world, and the correction of partial or incorrect understandings caused by prior experience (Duffy & Jonassen, 1991).

Objectivist education lends itself well to politically conservative purposes. Treating students as passive recipients who can be managed and controlled is consistent with viewing the aims of education as transmission of knowledge. Moreover, the objectivist model lends itself to the development of clear-cut instructional methods with precise strategies for teaching and testing learned material. The school-as-factory metaphor may offend most educators, but the efficiency and control that factories have over production processes is something that schools often envy.

Teaching Strategies Consistent With Vygotsky's Theory of Mind

Instructional methods based on Vygotsky solve some of the problems by emphasizing the need for social interaction and expert guidance within the *zone of proximal development* (Vygotsky, 1978; Newman, Griffin, & Cole, 1989). Vygotsky posited the zone of proximal development as a condition wherein students perform challenging tasks with support from other competent people. By taking part in group activities, individual learners internalize the goals and methods of more expert problem solvers. Vygotsky's zone of proximal development is consistent with cognitive conceptions of motivation. For example, Brophy (1987) suggests that the effort expended by the student on a task is the product: expectancy x value. Enriched problem-solving environments provide students with appropriate levels of challenge and difficulty, linkages between effort and outcome, and the teaching of goal-setting as ways to enhance expectancy. Value enhancement results from opportunities to make choices and decisions, modeling of interest from others, and opportunities for active response.

Unlike behaviorist approaches, Vygotsky-influenced approaches cannot be reduced down to a procedural recipe. Theories cannot be unambiguously operationalized into a set of rules. Of course, prescribing specific instructional strategies is something that instructional design-as well as mathematics education-has never backed away from. Current efforts in both fields are aiming to supply teachers with that missing ingredient they are looking for-insights on how to implement alternative models of mind in a practical way within the constraints of present school and societal structures.

Vygotsky contributed several insights that are relevant to mathematics educators; many of these insights are highlighted in other articles in this issue. Before discussing instructional strategies, we would like to briefly comment on a few of Vygotsky's original ideas:

1. *Primacy of the social*. Vygotsky's claim that cognition is the internalization of social interaction is a powerful idea. Some critics believe that individualist psychology has dominated our study of human behavior for too long (Faulconer & Williams, 1990). Following this thinking, we should de-throne individual cognition as the central foundation for education and re-acknowledge the important role of social and group dynamics in student development. Vygotsky's theory of social cognition reinforces a new social/cultural foundation for understanding education processes.

2. *Motivation and attitude development*. A social/cultural approach to cognition provides a fresh and much needed slant on questions of attitude development and motivation. Traditional cognitive psychology artificially separates cognition from affect, resulting in the general de-valuing of attitudes at the expense of skill acquisition (Lepper, 1988).

Vygotsky's theory of mind makes questions of affect, motivation, and will centrally (and not peripherally) important in understanding educational processes.

3. *The role of dialogue.* Dialogue-the two-way, interactive exchange between two speakers-is naturally conceived in terms of human conversation. In the 1930s, Vygotsky highlighted the dialogue that occurs between mother and child, or between teacher and student. Today, student-technology interactions are similar to these natural interactions in some ways that are important for learning. We would not suggest, however, that human-human dialogues are qualitatively identical to human-technology dialogues; important differences do exist and should be studied. Our point is that modern learning technologies suggest that the term's meaning now can be extended to include interactions between learners and technology-based tools and agents.

4. *The zone of proximal development*. The zone of proximal development was conceived of in terms of the added capacity a child has when supported in performance by by a teacher or more skilled peers. The zone of proximal development is that "distance" between a child's unassisted capability and that child's capability to perform with support. The child's zone can be "bridged" by opportunities for practice with external support. While acknowledging the primacy of social modeling and support, we would again extend methods of bridging the zone to include technological tools and devices. Technology has afforded modeling and simulation of a whole range of mental and natural processes; computer-based environments and tools can provide both context and support for meaningful problem-solving activities. This broadened view of bridging the zone of proximal development seems to be consistent with Vygotsky's emphasis on human-tool interaction.

With these distinctions made, we review below several teaching strategies that are consistent with a Vygotskian perspective.

Cognitive Apprenticeship

Following Vygotsky's and Dewey's (1933) lead, Brown, Collins & Duguid (1989) see conceptual knowledge as being similar to a set of tools. Often, students are asked to use the tools of a discipline (mathematical formulas, for example) without being able to adopt its culture. Collins and his colleagues identified Schoenfeld's (1985) approach to teaching college math as an example of a cognitive apprenticeship. In apprenticeship learning, skills are learned in the community of practitioners through observation, coaching and successive approximation (practice). After observing an expert execute an activity (modeling), the learner tries it with teacher guidance (coaching). The expert provides reminders ("scaffolding") which are removed (fading) once the task can be approximated. Sometimes the social context in which apprenticeship takes place includes a variety of expert models and other learners (Wertsch, 1985; Rogoff & Lave, 1984). Exposure to multiple ways of accomplishing a task and varying degrees of skill helps the learner recognize that there is no one embodiment of expertise and encourages them to view learning as a continuing process (Vygotsky, 1978).

Cognitive apprenticeships bear some resemblance to Vygotsky's notion of the zone of proximal development (Moll, 1990; Newman, Griffin, & Cole, 1989). In both cases, the intent is to engage the learner in meaningful, constructive activities that encourage growth and tryout of new conceptions and skills. Vygotsky's emphasis was on social interactions wherein teachers and peers can help learners bridge the zone of proximal development; Collins (1991) emphasizes the role technology can play in creating learning environments and performance supports.

Anchored Instruction

The Cognition and Technology Group at Vanderbilt (1990) has developed a promising instructional strategy that they call anchored instruction. Problem-solving environments for cooperative learning are *anchored* in videodisc-based, information-rich stories that tell a story and set up a problem to be solved. The random-access capability of this technology allows teachers and students to non-linearly search for embedded data and clues for the solution of problems for extended periods of time. The *Jasper* series of problem-based instructional lessons are all based on videodisc "macro-contexts" for problem-solving activities. These projects were shown to help students sustain thinking, find and define problems, generate subgoals and subproblems, and transfer knowledge outside the classroom.

Anchored instruction builds upon several aspects of Vygotsky's principles into instruction. The explicit problem statement within the materials provides a clear focus for students' cooperative activities. The rich learning environment made possible by the use of a dynamic, visual and spatial format allows exploration from multiple perspectives, and provides students with the opportunity to participate in high-quality discussions. The role of the teacher as participant as well as guide helps the student learn to solve problems through opportunities to work with an expert (Vygotsky, 1978). *Jasper's* story-based problem-solving approach has been shown to develop integrated knowledge structures that transfer to more complex tasks.

Learning Environments

Farnham-Diggory (1992) traces interest in learning environments back to Rousseau's *Emile*. In like manner, John Dewey's progressive education movement depended on rich, authentic learning environments. Following Vygotsky, two keys to successful instruction are: (1) the richness of the learning environment and (2) the quality of the social support and interaction. Learning environments are important elements of instruction, but students cannot be thrown into complex settings and left to themselves. They need the kind of help that comes from interaction with peers, with a knowledgeable teacher, and with relevant tools and information. There is thus a continuing dialectic or tension between student-initiated exploration and teacher guidance.

Technology-based learning environments allow users to interact with complex systems having multiple interrelated dynamic variables. Simulations allow for the modeling of complex, authentic environments. Future developments such as real-time video (similar to *QuickTime*, for the Macintosh) and natural language interfaces will enhance the efficiency and realism of these learning environments. Computer-based learning environments can be coupled with varying degrees of overt guidance and performance support, depending on the needs of the learner.

Conclusion

A major goal of all instructional design is to make a lasting impression on the learner. A purportedly ancient Chinese proverb posits:

"I hear and I forget

I see and I remember

I do and I understand"

Though oversimplified, the popularity of this aphorism suggests there is something to it. What is it we would like to accomplish with problem-solving activities in our schools? Employers and society in general would probably agree that self-confident, adaptive, conscientious graduates (high school, Bachelor, PhD, etc.) with the ability to apply knowledge in a variety of situations would be an appropriate goal. Recent developments in cognitive science combined with developing technologies applied to education now make it feasible to achieve this goal by re-creating important aspects of the world-at-large within the classroom. As we continue to learn more about effective instructional design, we need to expect continuing change in schools and classrooms as educators test out and apply their knowledge to improve student learning.

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