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Making sense of rate of change: Secondary students' reasoning about changing quantities

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MAKING SENSE OF RATE OF CHANGE: SECONDARY STUDENTS' REASONING ABOUT CHANGING QUANTITIES

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ABSTRACT

Rate of change is an important concept for students to study, and little is known about the ways in which secondary students make sense of rate of change. In this qualitative study I examined how four high school students who have not taken calculus reasoned about changing quantities when interacting with mathematical tasks involving multiple representations of constant and varying rates of change. Employing a cognitive perspective on mathematical reasoning, I conducted a series of five individual, task-based interviews with each student. Although mathematical reasoning is not directly observable, I made inferences regarding students' reasoning based on their explanations, their written work, and their gestures, counting as data those episodes during which students provided evidence of attending to changing quantities. Using the method of constant comparison, I traced students' reasoning from their explanations, written work and gestures to develop characterizations of their reasoning. In addition, I examined how students combined covariational (Carlson et al., 2002), transformational (Simon, 1996), and proportional (Lamon, 2007) reasoning when reasoning quantitatively about specific and nonspecific amounts of change. During the series of task-based interviews, students encountered the same set of tasks, but each of the students reasoned in different ways. The students utilized different combinations of covariational, transformational, and proportional reasoning when reasoning quantitatively about specific and nonspecific amounts of change. Interestingly, students' ways of reasoning and combinations of different types of reasoning remained consistent across the tasks and interviews, even though the tasks involved different contexts and different types of representations of mathematical quantities and

relationships between quantities. Students' combination of different forms of reasoning when reasoning about rate of change is notable because it gives insight into how students make sense of changing quantities involved in rates of change.